



ISSN 1814-5566 print
ISSN 1993-3517 online

МЕТАЛЕВІ КОНСТРУКЦІЇ
МЕТАЛЛИЧЕСКИЕ КОНСТРУКЦИИ
METAL CONSTRUCTIONS

2013, ТОМ 19, НОМЕР 1, 59–66
УДК 624.21:624.072.327

(13)-0283-2

УДОСКОНАЛЕННЯ РЕГУлювання ЗУСИЛЬ У ЗВОРОТНОМУ ВАНТОВОМУ МОСТУ ПРИ НЕЛІНІЙНОМУ МОДЕлюванні

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Отримана 7 лютого 2013; прийнята 22 лютого 2013.

Анотація. Незвичайний вантовий міст, у якому ванти розташовані під настилом моста і мають внаслідок зміни довжини стояків полігональне компонування, називається як зворотний вантовий міст (ЗВМ). Такі мости, як правило, мають чітко виражену нелінійність поведінки через приєднання вантів-затяжок до балки жорсткості. Ця нелінійність головним чином залежить від безрозмірного параметра гнучкості балки. Наведено проектування переднапружененої вантами ЗВМ конструкції з більш повним використанням поперечного перерізу. У статті надані нові можливості регулювання зусиль з використанням позацентрового приєднання вантів-затяжок. На прикладі численного експерименту показана ефективність запропонованої методики. Крім того, проведено порівняльний аналіз лінійної і нелінійної поведінки ЗВМ конструкції. На основі порівняльного аналізу деякі висновки стосуються надійності і точності визначення нелінійної поведінки конструкції, зокрема з урахуванням усіх початкових недосконалостей.

Ключові слова: зворотний вантовий міст (ОВМ), нелінійний аналіз, недосконалості, регулювання зусиль, раціональний екцентриситет, раціональний момент, загальний фактор недосконалостей, однопрольотна система, стиснуто-зігнутий елемент.

УСОВЕРШЕНСТВОВАНИЕ РЕГУЛИРОВАНИЯ УСИЛИЙ В ОБРАТНОМ ВАНТОВОМ МОСТЕ ПРИ НЕЛИНЕЙНОМ МОДЕЛИРОВАНИИ

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Получена 7 февраля 2013; принята 22 февраля 2013.

Аннотация. Необычный вантовый мост, в котором ванты расположены под настилом моста и имеют вследствие изменения длины стоек полигональную компоновку, называется как обратный вантовый мост (ОВМ). Такие мосты, как правило, обладают явно выраженной нелинейностью поведения из-за присоединения вантов-затяжек к балке жесткости. Эта нелинейность главным образом зависит от безразмерного параметра гибкости балки. Приводится проектирование преднапряженной вантами ОВМ конструкции с более полным использованием поперечного сечения. В статье представляются новые возможности регулирования усилий с использованием внецентренного присоединения вантов-затяжек. На примере численного эксперимента показана эффективность предлагаемой методики. Кроме того, проведен сравнительный анализ линейного и нелинейного поведения ОВМ конструкции. На основе сравнительного анализа некоторые выводы относятся к надёжности и точности определения нелинейного поведения конструкции, в частности с учетом всех начальных несовершенств.

Ключевые слова: обратный вантовый мост (ОВМ), нелинейный анализ, несовершенства, регулирования усилий, рациональный эксцентризитет, рациональный момент, общий фактор несовершенств, однопролетная система, скжато-изгибающий элемент.

IMPROVEMENTS ON THE STRUCTURAL RESPONSE CONTROL OF UNCONVENTIONAL CABLE-STAYED BRIDGES BY NONLINEAR ANALYSIS MODELLING

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Received 7 February 2013; accepted 22 February 2013.

Abstract. An unusual cable-stayed bridge with the cable stays placed below the deck and having polygonal layout due to deviation by means of struts is named under-deck cable-stayed (UDCS) bridge. UDCS bridges usually have significant nonlinear behaviour due to direct anchoring of cable-staying system into the deck. These nonlinearities mostly depend on dimensionless slenderness parameter. Designing UDCS structures for the better cross-sections utilization the prestressing of cable stays is widely used. This paper presents a new perspective for structural response control by means of eccentric anchoring of cable-staying system. Numerical examples emphasises the efficiency of the proposed computational technique. Moreover, the comparison analysis carried out considering structural behaviour of the UDCS structure both as linear and nonlinear. On the basis of such comparative studies, several conclusions are drawn concerning the safety and accuracy of the structural behaviour estimates yielded by nonlinear analysis and, in particular, when accounting for all initial imperfections.

Keywords: UDCS bridge, nonlinear analysis, imperfections, structural response control, rational eccentricity, rational moment, generalized imperfections factor, simple-span system, beam-column.

1. Introduction

Under-deck cable-stayed (UDCS) bridge is a result of developments in form-finding and refers to a relatively new form of the cable-stayed bridge. The proposal of this new bridge type arose in Fritz Leonhardt's mind as a consequence of trying to solve a complex engineering problem [1]. In UDCS bridge, intermediate pylons are replaced by supporting system of cable stays and struts. Direct anchoring of under cable stays evokes axial response in the main girder of the bridge and when load is applied it behaves under interaction of bending and compression. Due to previous notice, this study considers the deck of the bridges as longitudinal girder by means of steel beam-column element. Issues addressed to the structural behavior of the UDCS structures concern mostly the externally prestressed concrete slabs or beams [2–5] (Menn and Gauvreau 1987, Virlogeux et al. 1994, Harajli et al. 1999, Aravinthan et al. 2005, Fürst and Marti 1999, Lou et al. 2012). The par-

ticular case of steel UDCS structures was mostly considered with the approach of roof beam-string structures [6–9]. There are several studies spread in literature conducted on the structural behaviour of the combined structures [10–13]. Moreover, a great attention has been paid for the steel structures optimization by performing design and structural behaviour analysis [14–16].

This paper presents a new point of view to the possibility to control structural response of the main steel girder of the UDCS bridge by means of eccentric anchoring of the cable-staying system. In particular, this study aims at providing a contribution to a better understanding of structural behavior and a more efficient analysis of UDCS bridges.

2. Proposed method

Very few studies have been conducted on an attempt to control structural response of the

UDCS structures. A new approach on the flexural response control of the simple-span bridges was proposed by Juozapaitis et al. [17]. The study was performed on the case of UDCS bridge reconstruction with no suspension during exploitation. The simple-span UDCS bridge with mono cable-staying system was considered as shown in Figure 1. Proposed technique relies on the approach of the eccentrically anchored cable-staying system, thus arousing the bending moments at the support sections of the main girder. For the simplified linear analysis of the bridge calculation model is made considering individual member of the main girder as shown in Figure 1. Due to direct anchoring of the cable-staying system and applied distributed transverse load the considering simply supported structural element refers to beam-column and simultaneous action of bending and compression must be considered.

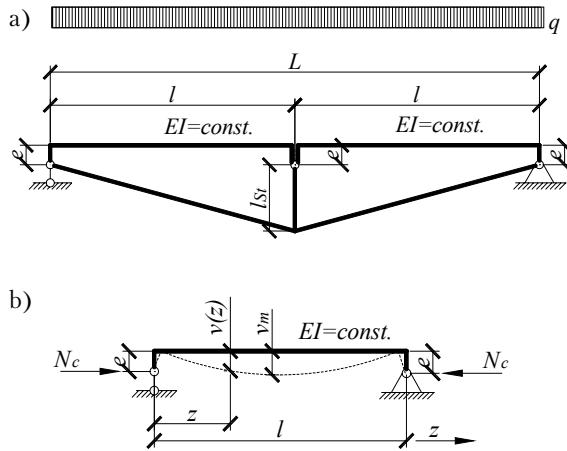


Figure 1. Calculation model of UDCS bridge: a) whole structure; b) individual member.

When the height of the cross-section of the girder is constant and assuming [17]:

$$|M(z=0)| = |M(z=L/2)|. \quad (1)$$

The rational eccentricity may be obtained as [17]:

$$e_{rac} = \frac{qL^2}{16N_c}. \quad (2)$$

and rational bending moment is equal to:

$$M_{rac} = \frac{qL^2}{16}. \quad (3)$$

Numerical evaluation

On the basis of previously presented study the numerical evaluation was performed to present the efficiency of the proposed technique. Figure 2 shows that with adopting rational eccentricity for the linear analysis of simple-span UDCS structure with mono cable-staying system the efficient distribution of bending moments can be achieved. By introducing hogging moments at the support sections of the girder the extreme values of the bending moments decrease twice compare to structure without structural response control. This advantage refers to increases in economy due to effective cross-section utilization.

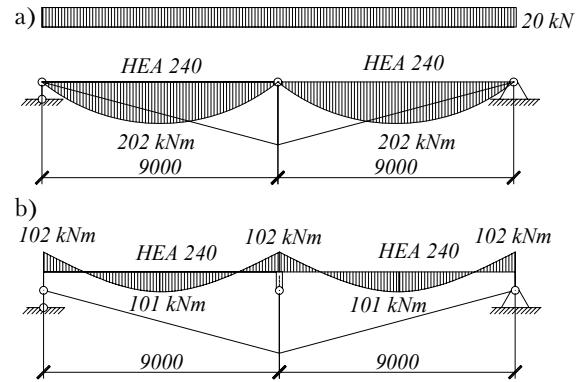


Figure 2. Bending moments diagram: a) without structural response control; b) with structural response control.

Moreover, the computer-aided analysis was performed using nonlinear solver ANSYS for structural behaviour analysis of the UDCS bridge. Analysis was carried out by varying slenderness parameter kl which expresses axial compression-flexural rigidity relationship:

$$kl = l \sqrt{\frac{N_c}{EI}}. \quad (4)$$

The axial response in UDCS bridge approximately may be accounted as:

$$N_c = \frac{qL^2}{8l_{st}}, \quad (5)$$

where L is span of the bridge and l_{st} is the length of the struts shown in Figure 1.

Table 1 shows that UDCS bridges have significant nonlinear behaviour which increases by increasing slenderness parameter kl . Thus the

Table 1. Results of the numerical analysis of the structural behaviour of the UDCS bridge

kl	q, kN/m	v _{linear} , mm	v _{nonlinear} , mm	v _{linear} / v _{nonlinear}	%	M _{max} , linear, kNm	M _{max} , nonlinear, kNm	M _{max,linear} /M _{max, nonlinear} %
1,64	20	117,1	174,1	48,61 %		202	279	28 %
1,07	20	54,0	62,0	14,87 %		203	230	12 %
0,95	20	47,2	52,1	10,31 %		202	223	9 %
0,79	20	32,1	37,2	16,20 %		203	216	6 %
0,60	20	21,2	21,9	3,42 %		202	210	4 %

attempt to adopt rational eccentricities obtained by Eq. 1 for the nonlinear analysis gives major disagreements as presented in Table 2. Last column of the Table 1 shows that linear analysis for the UDCS bridges can be reasonable for the values of the slenderness parameter less than one.

Table 2. Linear evaluation of the structural response control

kl	q, kN/ m	e _{rac} , mm	Linear	Non-linear
			M _{max} / M _{min} , %	M _{max} /M _{min} , %
1,64	20	188	-0,6 %	30,1 %
1,07	20	188	-0,5 %	10,9 %
0,95	20	188	-0,5 %	7,8 %
0,79	20	188	-0,5 %	5,1 %
0,60	20	188	-0,6 %	2,6 %

3. Nonlinear modelling

For accurate contemplation of UDCS structures the formula for the rational eccentricity derived accounting for the initial imperfections and con-

sidering the equilibrium of beam-column element shown in Figure 3 by approach of moderately large displacement theory. On the basis of previous mentioned theory, the structure is in its deformed state thus the second order effects are accounted.

When considering simultaneous action of bending and compression, the deflection at z is increased by deflection and the differential equation of bending becomes:

$$-Elv(z)^{'''} - N_0(v(z) + v_0(z))' + N_0e + q = 0. \quad (6)$$

Adopting a sinusoidal function for an initial geometrical imperfection:

$$v_0(z) = v_{m0} \sin \frac{\pi z}{l} \quad (7)$$

and introducing:

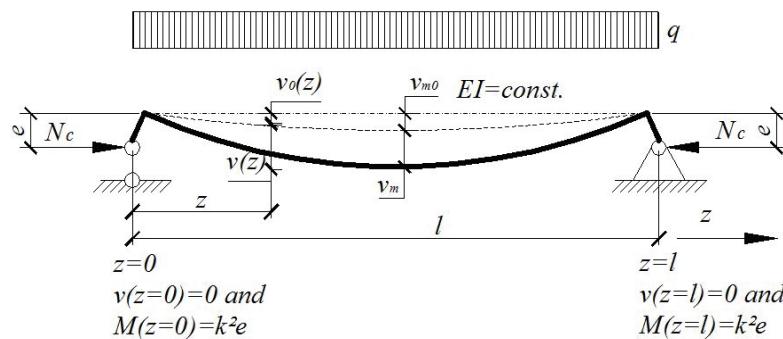
$$k^2 = \frac{N_c}{EI}. \quad (8)$$

The Eq. 6 may be rewritten as:

$$v(z)''' + k^2(v(z)'' + v_0(z)'' - e) - \frac{q}{EI} = 0. \quad (9)$$

The arbitrary solution of the Eq. 3 is:

$$v(z)C_1 \sin kz + C_2 \cos kz + C_3 z + C_4 + v_{part} = 0. \quad (10)$$

**Figure 3.** Calculation model of individual member of the girder.

When the transverse distributed load is assumed to be constant the particular solution can be written as:

$$v_{part} = \frac{v_{m0} \sin \pi z/l}{(\pi^2/(kl)^2) - 1} + e + \frac{qz^2}{2EIk^2}. \quad (11)$$

For the sake of simplification introduce:

$$n = \sqrt{\pi^2/(kl)^2 - 1}. \quad (12)$$

By combining Eq. 10 with the boundary conditions shown in Figure 2, the bending moment at any point of the girder may be obtained as:

$$\begin{aligned} M(z) = & \left(\frac{ql^2}{(kl)^2} - \frac{Ele(kl)^2}{l^2} \right) \times \\ & \times \left(\tan \frac{kl}{2} \sin kz + \cos kz \right) - \\ & - \frac{ql^2}{(kl)^2} + \frac{\pi^2}{l^2} Elnv_{m0} \sin \frac{\pi z}{l}. \end{aligned} \quad (13)$$

Assuming the condition written by the Eq. 1 the rational eccentricity for the nonlinear modelling is:

$$e_{rac} = \frac{\pi^2 Elnv_{m0} \cos \frac{kl}{2}}{[(kl)^2(1+\cos \frac{kl}{2})]} + \frac{ql^4 (+\cos \frac{kl}{2})}{[EI(kl)^4(1+\cos \frac{kl}{2})]}. \quad (14)$$

Thus, the rational bending moment:

$$\begin{aligned} M_{rac} = & \frac{\pi^2 Elnv_{m0}}{l^2} - q(\cos \frac{kl}{2} - 1) + \\ & + \frac{Ee_{rac}(kl)^2}{l^2 \cos \frac{kl}{2}}. \end{aligned} \quad (15)$$

For the accounting of all initial imperfections the concept of generalized imperfections factor applied. The generalized imperfection factor incorporates into account all the effects of initial

imperfections can be found in a real element for flexural buckling such as: lack of straightness, eccentricity of applied loads and residual stresses.

If consider the equivalent initial deflection associated to the generalized imperfection factor model prepared by Maquoi-Ronald for EC3, the magnitude of the initial deformed configuration is:

$$v_{m0} = a(\bar{\lambda} - 0,2)W/A, \quad (16)$$

where a is imperfection factor referring to column strength curves; $\bar{\lambda}$ is dimensionless slenderness; W and A are section modulus and cross-section area respectively.

For the case of small deflection analysis and accounting for initial imperfections the Eqs. 2 and 3 have to be extended such as:

$$e_{rac} = \frac{ql^2}{16N_c} + 0,5v_{m0} \quad (17)$$

and

$$M_{rac} = \frac{ql^2}{16} + 0,5v_{m0}N_c. \quad (18)$$

4. Numerical analysis

Considering UDCS bridge of the span 18,0 m shown in Figure 1 which is subjected to an axial force by direct anchoring of cable-staying system into the main girder of the bridge. The estimates of the structural response are carried out with the variation of the slenderness parameter for both linear and non-linear modelling. The transverse distributed load is assumed to be constant with the numerical value of 20 kN/m.

Table 3 presents the results of the numerical evaluation of the proposed structural response technique.

Table 3. Linear and nonlinear evaluation of the structural response technique

kl	v _{m0}	q, kN/m	Linear	Non- linear	Linear	Non- linear	Linear	Non- linear	Linear	Non- linear
			e _{rac} , mm	e _{rac} , mm	v, mm	v, mm	M _{rac} , kNm	M _{rac} , kNm	M _{max} / M _{min} , %	M _{max} /M min, %
1,64	24,8	20	200	224	47,0	51,8	108	121	0,0 %	-1,1 %
1,07	23,4	20	199	209	24,1	24,8	108	113	0,0 %	0,0 %
0,95	12,3	20	194	201	22,9	23,6	105	109	0,0 %	-0,2 %
0,79	11,5	20	193	198	18,5	18,9	104	107	0,0 %	-0,6 %
0,60	12,5	20	194	197	11,6	11,7	105	106	0,0 %	0,0 %

It is obvious that by increasing slenderness parameter, for this particular case decreasing flexural rigidity of the main girder, increases the nonlinearities in structural behaviour of the bridge. Table 3 shows that with adopting appropriate rational eccentricity obtained either by linear or nonlinear approach the effective distribution of bending moments can be achieved.

5. Conclusions

This paper details the effective method for the structural response control of the unconventional cable-stayed bridges with an emphasis on their significant nonlinear behaviour. Proposed method based on the eccentric anchoring of cable-staying system rather than application of pretension of cable stays as usually.

First, the methodology of the proposed method is discussed for the linear assessment of simple-span UNDS bridge. The base assumption for the structural response control has been developed and equations for the rational eccentricity and bending moment derived.

Secondly, an example is provided that illustrates the efficiency of the proposed method. Relevant achievements in bending moment's distribution can be obtained by adopting rational

eccentricity. The eccentric anchoring of cable-staying system evokes hogging moments at the support sections of the girder and by base assumption of the equalization of the extreme values of bending moments significant improvements on the cross-sections utilization may be achieved.

Thirdly, direct anchoring of cable-staying system into the main girder evokes axial forces and when transverse distributed load is applied the UNDS bridge has been considered under simultaneous action of bending and compression. Axial response of the main girder refers to the possibility of nonlinearities, thus the numerical analysis under variation of the slenderness parameter has been performed. The results showed that with the values of slenderness parameter greater than one considering structure performs in the significant nonlinear behaviour and application of rational eccentricity obtained by linear modelling is inappropriate.

Lastly, the paper presents a basic extension of the proposed structural response control method by nonlinear modelling that accounts for all initial imperfections. This improvement lets to achieve proficient utilization of cross-sections and efficient distribution of bending moments by application of accurate and appropriate technique.

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