



ISSN 1814-5566 print

ISSN 1993-3517 online

**МЕТАЛЕВІ КОНСТРУКЦІЇ**  
**МЕТАЛЛИЧЕСКИЕ КОНСТРУКЦИИ**  
**METAL CONSTRUCTIONS**

2014, ТОМ 20, НОМЕР 4, 209–220

УДК 624.074:042

(14)-0320-2

## **УТОЧНЕНІ РОЗРАХУНКОВІ МОДЕЛІ ДЛЯ ДОСЛІДЖЕННЯ СТІЙКОСТІ ОПОРНОГО КОНТУРА МЕМБРАННОГО ПОКРИТТЯ НА КВАДРАТНОМУ І ЕЛІПТИЧНОМУ ПЛАНАХ**

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*Отримана 3 листопада 2014; прийнята 26 грудня 2014.*

**Анотація.** У статті розглянуто проблемні питання дослідження стійкості опорного контура, які розцінюються як загальні для мембранних покриттів на прямокутному і еліптичному планах і є необхідною стадією при розробці проектних рішень. Розроблено методичний підхід, що дозволяє оцінити можливість втрати стійкості опорного контура мембранного покриття. Визначено деформаційні критерії для об'єктів-аналогів, для чого на конкретних прикладах розглянуто чисельне моделювання процесу втрати стійкості центрально та позацентрово стиснутих стрижнів, а також кругового і еліптичного кілець. Проаналізовано розрахункові значення напружень для конкретних варіантів реалізації мембранних покриттів. Отримані рішення дозволяють уточнити і оцінити значення критичної сили для елементів опорних контурів мембранних покриттів на прямокутному і еліптичному планах.

**Ключові слова:** мембранне покриття, стиснено-зігнутий опорний контур, втрата стійкості, деформаційні критерії, об'єкти-аналоги.

## **УТОЧНЕННЫЕ РАСЧЕТНЫЕ МОДЕЛИ ДЛЯ ИССЛЕДОВАНИЯ УСТОЙЧИВОСТИ ОПОРНОГО КОНТУРА МЕМБРАННОГО ПОКРЫТИЯ НА КВАДРАТНОМ И ЭЛЛИПТИЧЕСКОМ ПЛАНАХ**

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*Получена 3 ноября 2014; принята 26 декабря 2014.*

**Аннотация.** В статье рассмотрены проблемные вопросы исследования устойчивости опорного контура, которые расцениваются как общие для мембранных покрытий на прямоугольном и эллиптическом планах и являются необходимой стадией при разработке проектных решений. Разработан методический подход, позволяющий оценить возможность потери устойчивости опорного контура мембранного покрытия. Определены деформационные критерии для объектов-аналогов, для чего на конкретных примерах рассмотрено численное моделирование процесса потери устойчивости центрально и внецентренно сжатых стержней, а также кругового и эллиптического колец. Проанализированы расчетные значения напряжений для конкретных вариантов реализации мембранных покрытий. Полученные решения позволяют уточнить и оценить значения критической силы для элементов опорных контуров мембранных покрытий на прямоугольном и эллиптическом планах.

**Ключевые слова:** мембранное покрытие, сжато-изогнутый опорный контур, потеря устойчивости, деформационные критерии, объекты-аналоги.

## REFINED DESIGN MODELS FOR INVESTIGATING THE STABILITY OF THE SUPPORT CONTOUR MEMBRANE ROOF ON THE SQUARE AND ELLIPTICAL PLANS

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*Received 3 November 2013; accepted 26 December 2014.*

**Abstract.** The article deals with the problematic issues of investigating the stability of the reference circuit, which are regarded as common to membrane roofs on rectangular and elliptical plans and are a necessary step in the development of design solutions. Methodical approach for assessing the possibility of loss of stability of the support contour membrane roof has been developed. Criteria for deformation of objects-analogues have been determined, the numerical simulation of the buckling centrally and eccentrically compressed bars, as well as circular and elliptical rings, has been considered for specific example. Values of the calculated stresses for specific embodiments of the membrane roofs have been analyzed. The resulting solutions can specify and estimate the values of the critical force for the elements of the support contour membrane roofs on rectangular and elliptical plans.

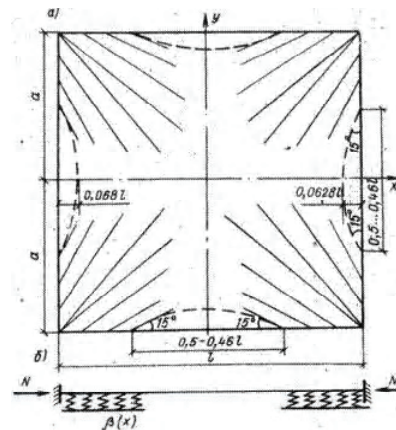
**Keywords:** membrane roofs, compressed-bent support contour, loss of stability, deformation criteria, objects-analogues.

### Introduction

Most of the work is devoted to the study of the deformation and the development of methods for the analysis of membrane roofs. However, some issues, such as the behavior of the flight membrane roof (for example, if a thin sheet receives compressive stress) as well as the strength and stability of the compressed-bent contour remain poorly understood. The most well-known approaches to the study of the above problems are theoretical and experimental studies conducted under the leadership of V. I. Trofimov and P. G. Eremeev [5, 7]. Of the strength and development of methods for the design of membrane roof on a square and round plans are sufficiently well-developed procedure.

As well as L. I. Goldenberg's works [2, 3] in which there were outlined approaches to study stability of compressed-bent support contours of sagged membrane roofs on the square plan. Given the presence of compressive stresses in the membrane of marginal zone of the shell (Fig. 1), which can not be perceived by a thin sheet without buckling, as a basis for the design model put forward by Goldenberg, there was suggested an idea of presentation of a compressed-bent rectilinear bar (an

element of the support contour) in the form of beam on the elastic foundation in the form of a tension membrane continuously connected with the support contour along its length. As a criterion of taking a sheet membrane off the work is the formation of a biaxial compression zone on the membrane local part, in its turn that makes it impossible to perform supporting functions of the elastic foundation.



**Figure 1.** Experimental picture of the stress-strain state of the membrane of near-contour zone (a) and the design scheme of contour edges (b).

A similar phenomenon can be observed not only in the membrane roof on a rectangular, but the elliptic plane. In its turn the length of the membrane part taken off the work determines a free length of the part of the compressed-bent element of the support contour which might suffer of collapse.

As can be seen from Fig. 1, a middle part of the contour rid at the length of about  $0.5 l$  contacts with that part of the membrane which lost its stability because of compressive stresses. The authors suggest determining an axial critical load by the following equations:

$$N_{cr} = \left( \frac{-0,0283 \cdot w^2}{f_k} + \frac{0,126 \cdot c \cdot a}{f_k} + \frac{0,00942 \cdot \Delta \cdot a}{f_k} \right) \cdot E_0 \cdot t + \frac{\pi^2 E_c I}{a^2}, \quad (1)$$

$$N_{cr} = [\alpha(0,194 + 0,423n + 5,08k + 3,78kn) - (0,15 + 0,342n + 4,44k + 6kn)] \times \frac{w^2 E_0 \cdot t}{f_k T} + \frac{\pi^2 E_c I}{a^2}, \quad (2)$$

$w$  – the sag of the membrane;  
 $f_k$  – the sag of the contour edges horizontally;  
 $c, \Delta, T$  – quantities depending on the size, the sag of the membrane, the longitudinal and bending rigidity of the support contour;  
 $a$  – a half of a roof span;  
 $t$  – a membrane thickness;  
 $E_c I$  – a contour bending rigidity;  
 $E_0 = E/(1 - \mu^2)$ , where  $E$  – a modulus of elasticity,  $\mu$  – Poisson's ratio;  
 $\alpha = (a_B/a)^2$  – coefficient, where  $a_B$  – half the distance between the ends of vuty;  
 $n, k$  – stiffness characteristics.

In what follows the authors suggest testing the contour stability in the horizontal plane in accordance with the design norms [4] for an eccentrically loaded bar with eccentricity  $e = M/N$  with the design length factor

$$\mu = \frac{0,5\pi}{a} \sqrt{\frac{E_c I}{N_{cr}}}.$$

Despite a significance and primary importance of the results obtained, there are some questions which, to our mind, were beyond the investigations, among them are the following:

– for the roofs on the square and rectangular plans under consideration there is given a too

wide turndown of the basic design parameters, in the end that can result in creating roofs with obviously unpractical parameters;

– there was absolutely not studied the behavior of compressed-bent contours of sagged membrane roofs on the circular and elliptical plans in which under non-uniformly distributed loads there might form near-contour zones of a biaxial compression within which a membrane does not have a supporting effect in the form of an elastic basis for a compressed-bent contour. The above problematic issues can be considered as common to membrane roofs, but other than that, there are no plans for elliptic like engineering solutions, in accordance with which to evaluate the risk of loss of stability of the design.

In this regard, consider the simplest case the loss of stability in the elastic stage of the work material, and an analog problem to be solved for the membrane roof on a rectangular plan, consider the process of numerical simulation of the loss of stability centrally compressed and eccentric compression bars.

As one example, consider rack,  $l = 4$  m with initial geometric imperfections  $(1/700)l$ , loaded with a force  $F = 948$  kN. When finite element modeling on the length of the bar by additional nodes was divided into 20 equal parts. According to the results of geometrically nonlinear analysis plotted «F– $\Delta$ » (see Table 1, Scheme 1), which highlighted the criteria for assessing the strained state of the bar corresponding to the time of the loss of stability in the theoretical value of the critical force  $N_{cr} = 474$  kN:

- the relative deflection of the rod, which is equal to  $(1/63)l$ ;
- slope of the tangent dependence «F– $\Delta$ » at the point of the loss of stability (ie, the first derivative of the unknown function «F– $\Delta$ » at this point), equal to  $tg \alpha = 12,1$ .

As an example, the 2nd payment is made eccentrically compressed bar (estimated Scheme 2) with the following characteristics corresponding to its parameters of the stress-strain state and stiffness characteristics of the support contour membrane roof, shown in Fig. 2 and 3:

- the eccentricity of the application of the longitudinal force  $e = 0,24$  m,
- length of the bar  $l = 2a\mu = 21,6$  m,
- stiffness characteristics:  $EF = 7\,416\,000$  kN,  $EI = 480\,557$  kN·m<sup>2</sup>,

Table 1. To modeling of the loss of stability Central and eccentric compression bars

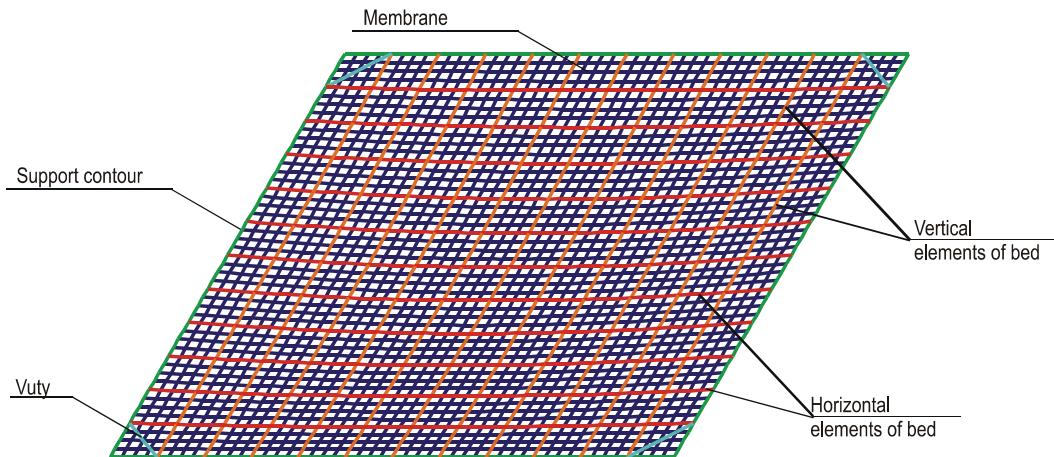
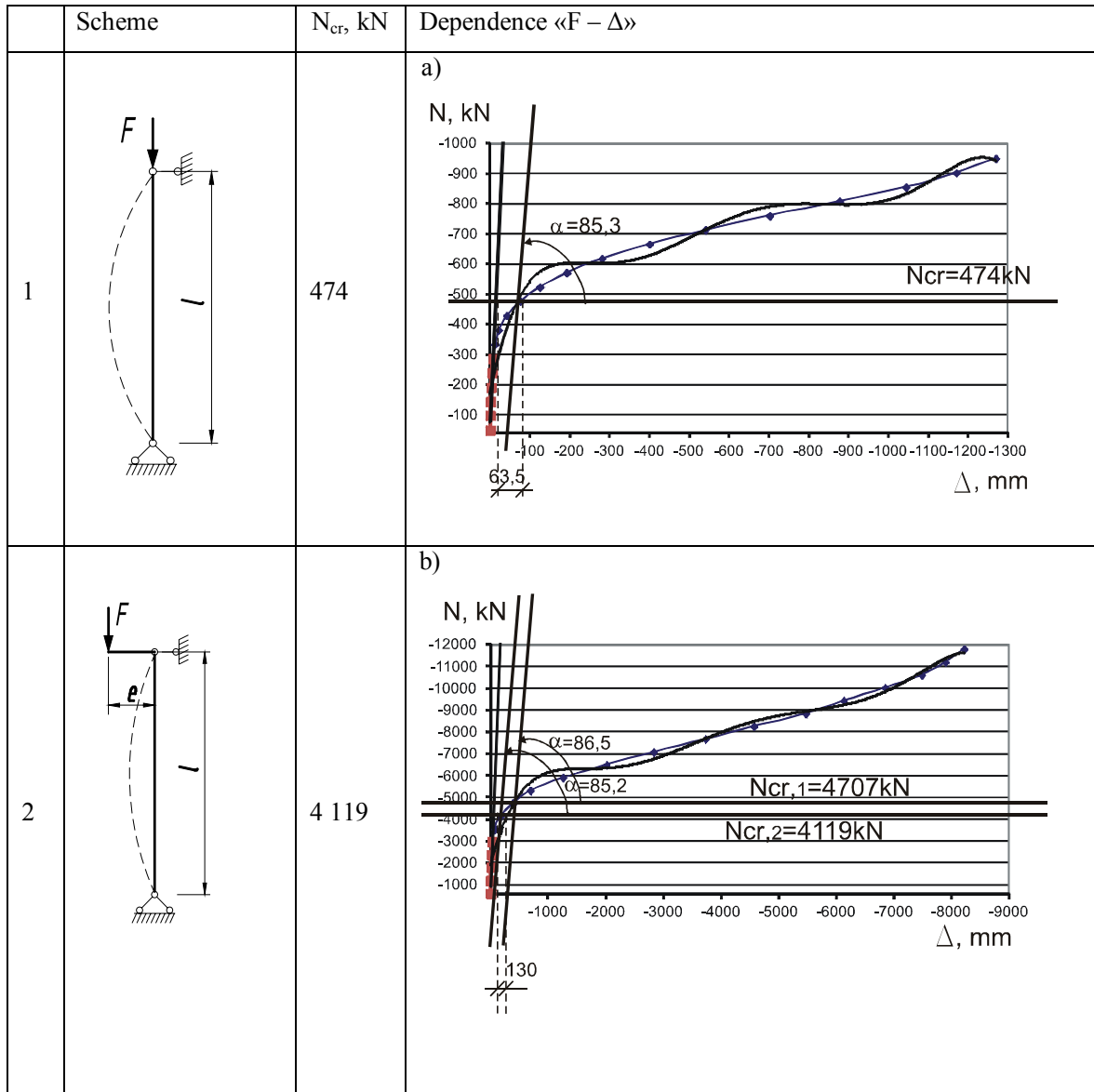
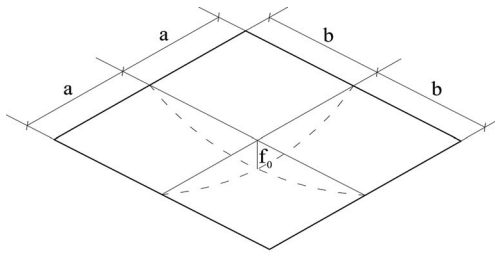


Figure 2. Constructive scheme of membrane roof on a square plan.



**Figure 3.** Geometric characteristics of sagging membrane roof on a square plan.

- the magnitude of the initial geometric imperfections  $(1/700)l$ .

The graph built for the circuit 2 based on the results of geometrically non-linear analysis for the above-mentioned deformation criteria identified two critical force values:

$N_{cr,1}$  – corresponds to the point where a sharp increase begins movement at equal loading steps  $(1/64,5)l$ ;

$N_{cr,2}$  – corresponds to the slope of the tangent corresponds to the beginning of the process the loss of stability ( $tg \alpha = 11,6$ , which is quite close to its value in the previous case  $tg \alpha = 12,1$ ), but the value is only relative displacements  $(1/166)l$ . Marked by an earlier onset the loss of stability at lower relative arching can be explained by an additional influence on the process of eccentric application of the compressive force.

As proposals for overcoming the outstanding problems assessing the sustainability of the support contour membrane roofs can be formulated in the following proposals:

- the use of finite element analysis helps to clarify some assumptions described above methods of calculating the stability of the support contour membrane roofs. The main disadvantage of the implemented approach with very little simplified and reasoned approach to determine  $l_0 = (0,92...1) a$ , is the use of a simplified idea that receives equally thin sheet membrane both compressive and tensile stresses. The incorrectness of this approach indicates a number of works (in particular the work of Vladimir Lensky [6]), where experimental and theoretically proved that the ability to perceive the compressive stress  $k_c = \sigma_c / \sigma_v$ , determined by the Gaussian curvature of the membrane shell (for plates loaded plane  $k_c = 0,15$ );

- count to tackle the design in 2-step procedure using the following algorithm:

- Stage 1:
  - holding the primary structure analysis of strength to rank the elements of near-contour zone of the membrane by 3 types (type 1 – two-side-tensiled, type 2 – compressed-stretched, type 3 – two-side-compressed). Moreover, all the membrane elements are considered to be equally receiving compressive force and tensile enveloped.
- Stage 2:
  - the formation of a design scheme for the calculation of the support contour for resistance using compressed elements in modeling membrane finite elements are used in the form of an orthotropic shell, which decrease the carrying capacity of elements in the direction of the compressed membrane accounted Reductions modulus  $E$  in this direction, as  $k_c E$ ,
  - in idealized design scheme is introduced initial deviation from straightness the support contour in the form of  $(1/700) L$  (where  $L$  – length of side support contour),
  - solve the problem of stress-strain state in geometrically nonlinear formulation using the method of incremental loading adjusted initial geometry of the structure at each step of loading,
  - fixing the time of a sharp increase in deformation (ie violations directly proportional to the « $F - \Delta$ »), set the value  $N_{cr}$ , mark the start bar the loss of stability.

Fig. 5 shows the load indicated by  $N$  horizontal displacement of the support contour  $\Delta$  for the following approximations membrane coating: 1 – idealized simulation of isotropic shell finite elements; 2 – Compressed modeling by finite elements of the membrane elements in the form of an orthotropic shell whose compressed direction on the bearing capacity of the membrane is adjusted as  $E' = 0,15E$ , 3 – the same as Scheme 2, but with an initial imperfection the support circuit  $(1/700) L$ .

As the object of the study design was chosen a membrane roof on a square plan with a predetermined sag arrow (see Fig. 2).

The parameters of the test membrane roof: the size of the semiaxes roof  $a=b=36$  m, the initial sag arrow  $f=1,6$  m, the membrane thickness  $t=2$  mm.

The initial shape of the surface covered by the host using the formula (3)

$$z = f_0 \left( 1 - \frac{x^n}{a^n} \right) \left( 1 - \frac{y^n}{b^n} \right), \quad (3)$$

where  $a, b$  – halves of the sides of a roof rectangular plan;  $f_0$  – an initial erecting sag in the center of a roof;  $x, y$  – running coordinates (Fig. 3).

The values for the stiffness characteristics of the support contour are taken on the basis of recommendations [7]  $\bar{\kappa} = 0,5, \bar{n} = 2,5 \cdot 10^{-5}$ .

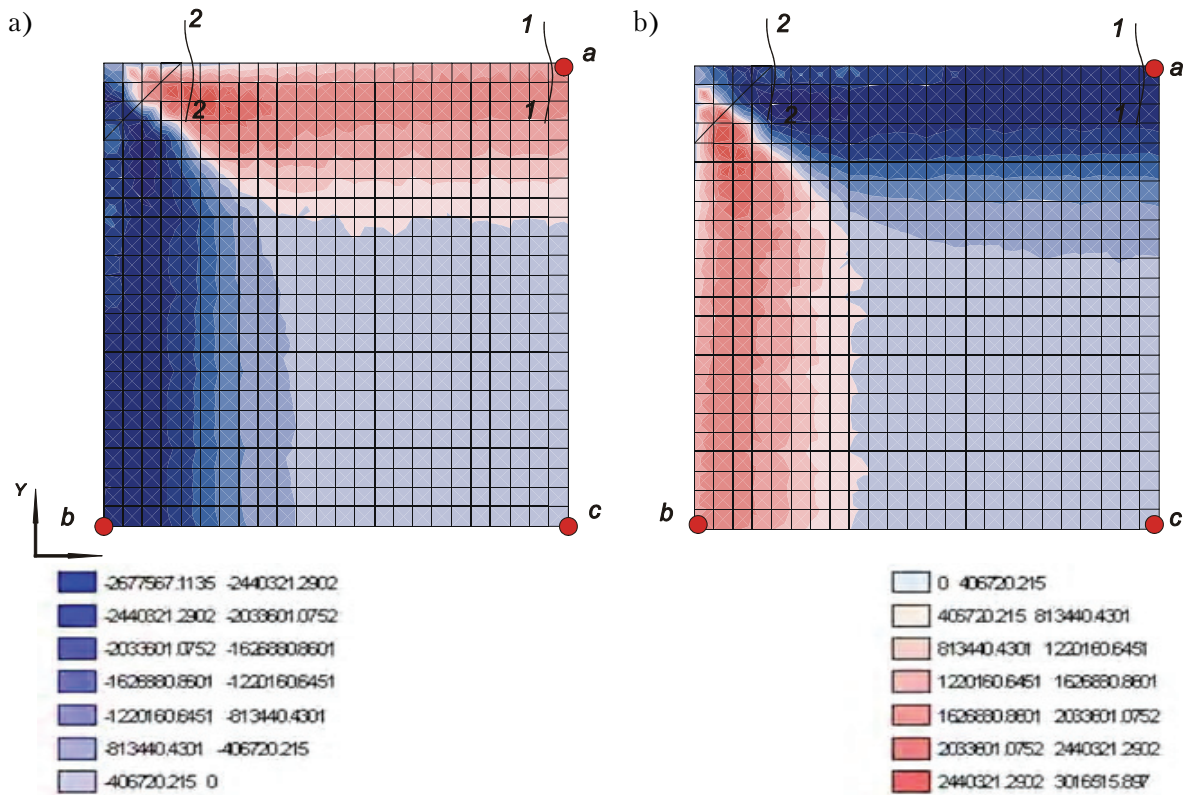
The numerical calculation was performed using the software package SCAD office. Consider a combination of loads: constant of its own weight and the time the snow ( $q_s = 1,5 \text{ kN/m}^2$ ). In the zone abutting the membrane shell to the support contour occurs biaxial stress state: in the  $y$  direction (see Figure 4) having a compressive stress, and in the perpendicular direction (along the  $x$  axis) and tensile stresses occur. In the middle zone the support contour portions are minor tensile stresses (as compared to the amount of compressive stresses), which

leads to a drastic reduction supporting effect of membrane relative to the support contour and creates the danger of the loss of stability in the horizontal plane of the contour in these sites.

If the considered design scheme membrane roof as criteria fixing the initial moment of buckling deformation criteria used for the same stiffness characteristics of the bar № 2 (see Table 1), we can obtain more accurate values of the critical force (compared to the results of the analytical calculation performed in the presence of simplifying set forth above, the prerequisites (see Table 2 and Figure 5).

The results of the calculation indicate a significant (40 %) differences from  $N_{cr} = -9\,933.3 \text{ kN}$ , calculated according to the formula (2). Increase the value of the critical force is due to partial inclusion membrane in collaboration with support contour, and thereby increase its load-bearing capacity.

Using a similar scheme of studies was considered a design scheme of the membrane roof on the elliptic plan (using the geometric parameters of the membrane roof the stadium «Olympic» in



**Figure 4.** Contour plots: a) tensile normal stresses  $\sigma_x$  in the horizontal direction; b) compressing the normal stress  $\sigma_y$  in the vertical direction;  $a$  and  $b$  – point with the greatest horizontal displacements of the support contour;  $c$  – point with the largest vertical deflection of the membrane; section 1–1 and 2–2 – the calculated cross section to test the strength of the support contour.



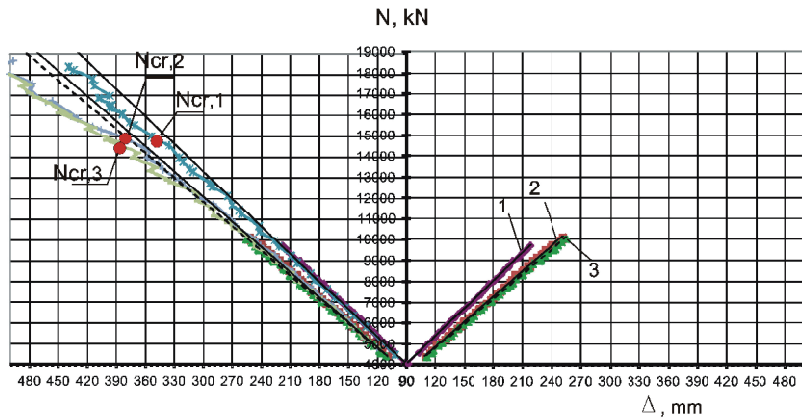
**Table 2.** The stress-strain state of the membrane roof on a square plan

Model	The maximum displacement, mm			Force in the support contour				Stresses in near-contour zone (elements in the cross section 1-1)	
				in the span (the cross section 1-1)		in the angular area (section 2-2)			
	x (τ. a)	y (τ. b)	z (τ. c)	N <sub>1</sub> , kN	M <sub>1</sub> , kN·m	N <sub>2</sub> , kN	M <sub>2</sub> , kN·m	N <sub>x</sub> , MPa	N <sub>y</sub> , MPa
1	223	218	1 785,6	-9 715,1	138,6	-4 589,0	0	1 657	-1 815
2	261	252	1 969,6	-10 103,3	162,3	-5 909,3	205,9	1 987	100,8
3	260	254	1 970,2	-10 026,0	164,9	-5 838,8	209,3	2 018	91,7

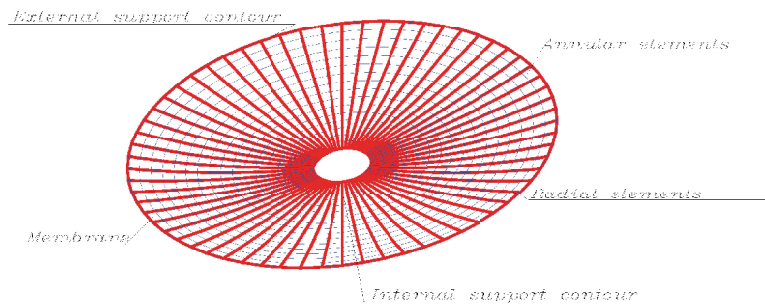
Where 1 – idealized isotropic; 2 – idealized orthotropic; 3 – orthotropic with initial imperfection,  $w_o$  – the deflection at the center of the membrane,  $f_k$  – full deflection contour edges in the horizontal plane.

Moscow with the size of the principal axes of 224 and 183 m). The construction is covered by the membrane shell attached supporting contour having a cross-sectional dimensions of 5 × 1,75 m, supported on steel columns. Sag in the center of the shell 12,5 m. Figure 6 shows a constructive solution of the test sagging membrane roof.

As mentioned above, for membrane roof on the elliptic plan are no engineering techniques to correctly assess the sustainability of compressed-bent support contour. Therefore, using the above approach, based on consideration of objects-analogues and determine the appropriate deformation criteria for roof on the elliptic plan as such an analogy,



**Figure 5.** The dependence of the longitudinal force in the support contour from its horizontal displacements «N-Δ».



**Figure 6.** Constructive scheme membrane roof on the elliptical plan.

consider the numerical simulation of the buckling of circular and elliptical rings with geometric and rigidity parameters «Olympic» and the initial imperfection  $(1/700) R$ , compressed radial load. Fixing deformation criteria in the form of relative displacement and the slope of the tangent holds for the critical load  $q_{cr} = 3EJ / R^3$  and the corresponding values  $N_{cr} = q_{cr} \cdot R$ . The calculation results are shown in Table 3. According to the results of calculation was plotted, which highlighted the criteria deformed state of the ring corresponding to the moment at the critical loss of stability force:

- the relative deflection of the bar (contour element), which is equal to  $1/680$ ;
- the slope of dependence «F – Δ» (ie, the first derivative at this point), equal to 2,45696.

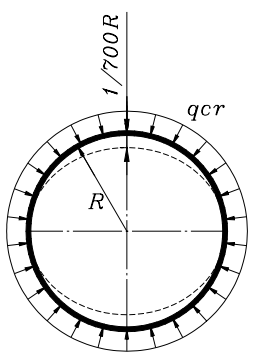
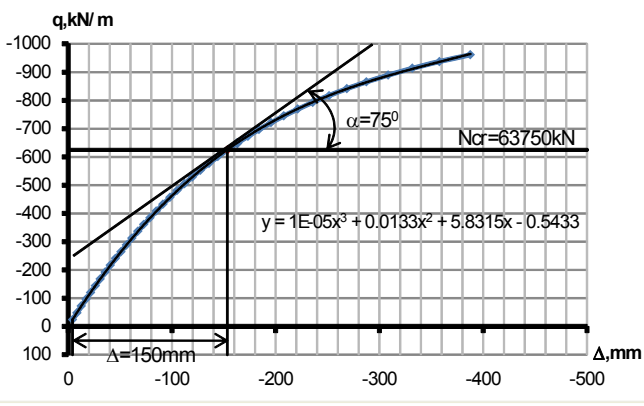
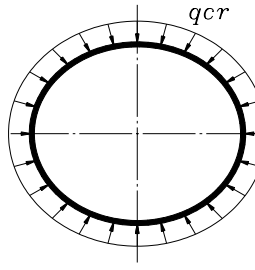
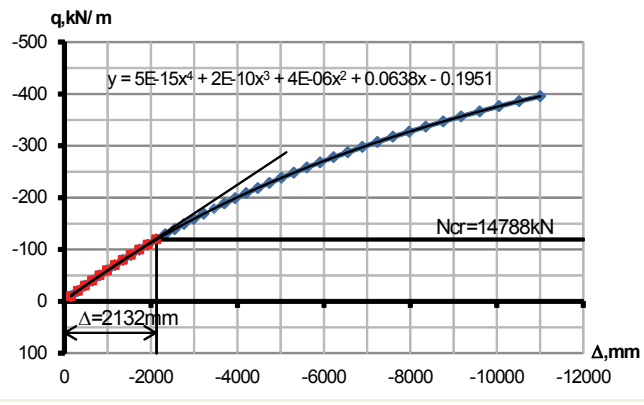
As the 2nd object-analog performed numerical simulation of loss of stability elliptical ring (Settlement Scheme 2) with rigidity characteristics corresponding to the support contour «Olympic» (longitudinal stiffness  $EF=1,099 \cdot 10^8$  kN, the flexu-

ral rigidity  $EI=3,41 \cdot 10^8$  kN·m<sup>2</sup>). Elliptical ring is compressed uniformly critical load  $q_{cr} = 536,5$  kN/m. On the graph recorded deformation criteria of loss of stability for the critical force  $N_{cr} = 14\,788$  kN – point of dish contour element (point deviation from the straight section). The calculation of the stress-strain state of the roof is also made for a 2-step procedure using in modeling orthotropic membrane shell finite elements to simulate the limited ability of perception of a thin sheet of compressive stress in the membrane of near-contour zone. The results of the calculation are shown in Fig. 7 and Table 4.

Fig. 7 shows that in the area when the project near-contour zone uniformly distributed load biaxial compression occurs over a length of 170 m.

Using the deformation criteria fixed schema 1 (Table 3), has allowed to establish the critical force under the action of uniformly distributed and skew-symmetric load on roof, characterized by different design combinations of  $N$  and  $M$  in the support contour (Fig. 8).

**Table 3.** Settlement schemes circular ring and an ellipse and a plot «F – Δ»

	Schema	$q_{cr}, kN/m$	Dependence «F – Δ»
1		964	
2		536.5	



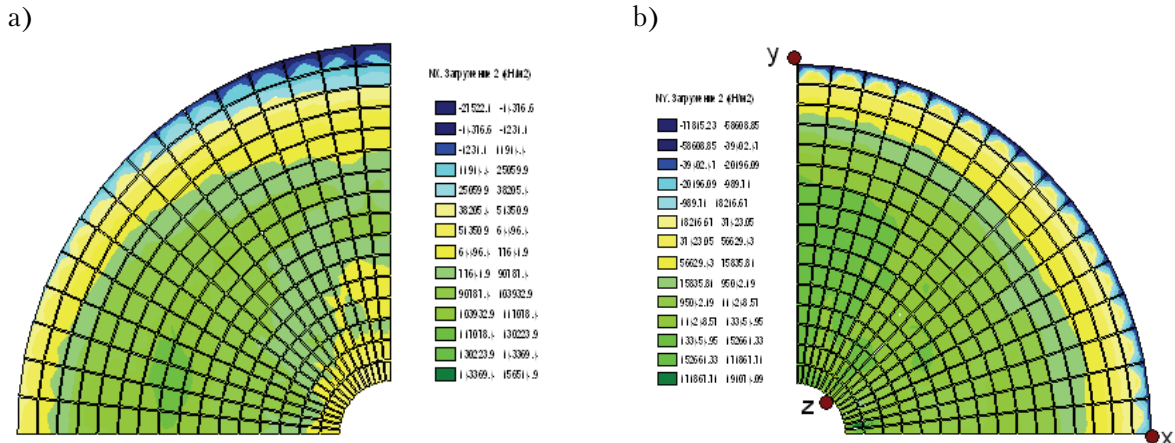


Figure 7. Contour plots a) radial stresses; b) hoop stress.

Table 4. Stress-strain state of the membrane roof on the elliptic plan

Model	The maximum displacement , mm			Force in the support contour		Stresses in near-contour zone	
	x	y	z	N, kN	M, kN·m	$N_p$ , MPa	$N_{Kz}$ , MPa
1	-33,59	-60,41	-397,52	-38 200,56	4 829,53	-27,522 1	-77,815 23
2	-40,91	-40,23	-490,30	-28 986,69	4 254,21	-33,515 52	-75,173 37
1*	-32,36	-68,36	-474,77	-40 391,20	5 025,26	7,734	-37,282
2*	-39,06	-44,04	-513,70	-30 204,32	4 105,46	3,625	-25,686

Note: 1 – idealized isotropic model for uniformly distributed load; 2 – an idealized model for the skew-symmetric load; 1\* – orthotropic model for uniformly distributed; 2\* – orthotropic model for skew-symmetric load.

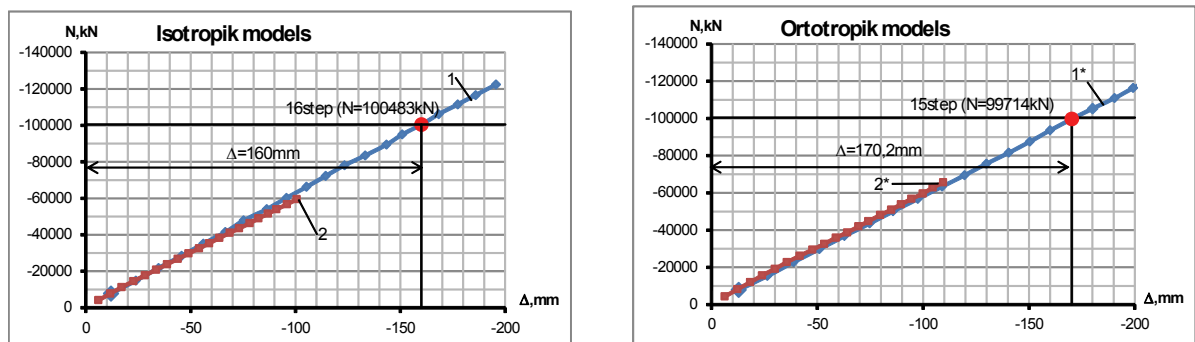


Figure 8. Dependence of a force from deformation of the parties supporting contour.

**Conclusions**

1. Developed a methodical approach that enables:  
 – with using clarifying the preconditions in the form of limited powers of perception sheet membrane compressive stresses in the near-contour zone,

– by establishing for object-analogs of deformation criteria, evaluate the possibility of loss of stability compressed-bent support contour membrane roof.  
 2. For membrane roof on a rectangular plan obtained solutions allow significantly (to 40 %)

to specify values  $N_{cr}$ . For membrane roof on the elliptic plan with the ratio of 1,0:1,2 for the first time made it possible to study to evaluate the value  $N_{cr}$  and set the priority options uniformly distributed load on the surface of the roof in the stability analysis of the support contour.

3. As an analysis of stresses formed when designing support contours to specific embodiments analyzed membrane roofs obtained conclusions

are valid only when using as material of contour high strength steels. In the case of low-carbon and low-alloy steels process loss of stability will occur in the elastic-plastic stage of the work of material, which will require performing the calculations in nonlinear elastic stage of the work material and the corresponding experimental verification of theoretical research.

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